A Study on Unblocked Reliability Assessing Road Network Operation Performance for Planning

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Abstract: Road network unblocked reliability can assess the operation performance of a road network at peak hour in order to support the decision-making involved in road network planning. Four forms of road network unblocked reliability (link, route, Origin-destination pair and whole network) are shown to assess the operation performance of each level of road network. This paper proposes a new methodology to calculate the unblocked reliability based on observed link flows under User Equilibrium. Based on real data, the proposed method is applied to assess a local road network. The unblocked reliability of the whole network is estimated to be 55% by calculation. This means that the operation performance is judged to lie in the medium congestion state. Moreover, link sensitivities are analyzed to find the important road section in which the reliability will be most increased if the road section is improved. The difficulty of unblocked reliability analysis based on observed link flows is to find a route choice pattern, i.e., which routes are selected and how many proportions of Origin-destination flows are loaded on these used routes between every Origin-destination pair. A new traffic assignment method is proposed based on the Frank-Wolf algorithm. This extended Frank-Wolf algorithm not only can find the link flows but also can solve the route choice pattern. As a result, the operation performance of the road network can be evaluated effectively at several levels of traffic demand, and the sensitive links for operation performance can be found in order to direct planning of a given road network.

Keywords: Operation Performance, Unblocked Reliability, User Equilibrium, Route Choice Pattern, Frank-Wolf Algorithm

1. INTRODUCTION

Road network reliability that can be considered as the route choice model has been proposed as a probability indicator. There are several reliability concepts in road network, such as connectivity reliability, travel time reliability\(^[3]\), capacity reliability\(^[2]\), and unblocked reliability, which latter is the probability of the road unit or system being able to maintain an unblocked state at peak hour\(^[3]\). Length of road is regarded as the factor influencing route choice behavior when unblocked reliability is applied to analyze investment optimization of road network\(^[4]\).

A methodology is proposed to calculate the road network unblocked reliability based on observed link flows. A method of sensitivity analysis is provided to find the important road sections in road network planning. These analyses consider the route choice analysis under User Equilibrium (UE). An extended Frank-Wolf algorithm is proposed to solve the route choice pattern which is the focal point for calculating unblocked reliability. Then, this method is applied to assess a local road network.

Notation

\(C\) capacity, [pcu/h]

\(f\) path flow, [pcu/h]

\(M\) set of used routes between Origin-destination (OD) pair

\(n\) number of traffic zones

\(p\) route choice proportion

\(q\) traffic demand between OD pair, [pcu/h]

\(R\) unblocked reliability

\(S\) sensitivity

\(T\) sum of each link’s travel time on studied path, [h]

\(t\) travel time, [h]

\(V\) traffic volume, [pcu/h]

\(x\) flow on a link, [pcu/h]

\(y\) auxiliary flow on a link, [pcu/h]

\(\alpha\) optimal proportion to minimize project function of mathematical program for UE

Subscripts

\(a\) link in the network

\(i\) origin node

\(j\) destination node

Superscripts

\(c\) counter for comparing calculated and observed link flows

\(e\) free-flow state

\(k\) iteration times

\(l\) shortest path between OD pair

\(m\) used route between OD pair

\(w\) number of paths between OD pair

2. UNBLOCKED RELIABILITY AND SENSITIVITY

2.1 Link unblocked reliability

The road operation situation is determined by the degree of congestion. The link unblocked reliability is 0% when the congestion degree, \(x_a/C_a\), reaches one, and 100% when the demand is zero. A curve that expresses
the relationship between reliability and congestion degree connects these two points. If the function \( f(x_c/C_a) \) means the unreliable probability of a link, the link unblocked is defined by the following equation:

\[
R_a = 1 - f(x_c/C_a) \tag{1}
\]

The function \( f(x_c/C_a) \) is defined as following function\(^5\):

\[
f(x_c/C_a) = \begin{cases} 
    x_c/C_a, & 0 \leq x_c/C_a \leq 1 \\
    1, & x_c/C_a > 1
\end{cases} \tag{2}
\]

2.2 Path unblocked reliability

A path is comprised of a link series which is one among several reasonable paths between an Origin-Destination (OD) pair \( ij \). Let path \( m \) consist of a set of links between origin \( i \) and destination \( j \). The link travel time, \( t_a(x_a) \), which represents the relationship between the flow and the travel time for link \( a \), is calculated using the standard Bureau of Public Road (BPR) function, as shown below:

\[
t_a(x_a) = t_a'(1 + 0.15(x_a/C_a)^3) \tag{3}
\]

If there are the same patterns of function for calculating the path’s travel time, the nominal degree of congestion on path \( m, V/C \), can be estimated by the following function:

\[
V/C = \left[ \left( T/T' - 1 \right) / 0.15 \right]^{25} \tag{4}
\]

The route unblocked reliability, \( R^m \), on route \( m \) connecting OD pair \( ij \) can be calculated similar to the link unblocked reliability as follows:

\[
R^m = \begin{cases} 
    1 - V/C, & 0 \leq V/C \leq 1 \\
    0, & V/C > 1
\end{cases} \tag{5}
\]

2.3 Origin-destination pair unblocked reliability

The reliability of an OD pair is regarded as the parallel combined structure system\(^6\). Between an OD pair, if the number of used routes is \( w \), the unblocked reliability of total routes between the OD pair is given by:

\[
R_{ij} = 1 - \prod_{m=1}^{w} (1 - R^m_{ij}) \tag{6}
\]

2.4 Unblocked reliability of whole road network

The unblocked reliability of the whole road network is defined as the average of total OD pairs in this network. There are \( n(n-1) \) OD pairs in a network including \( n \) traffic zones. The overall unblocked reliability, \( R \), is expressed as follows:

\[
R = \frac{1}{n(n-1)} \sum_{i=1}^{n} \sum_{j=i+1}^{n} R^m_{ij} \tag{7}
\]

2.5 Sensitivity analysis

Sensitivity analysis addresses what would have been the corresponding performance measure for the link reliability if a road network condition had been changed. The sensitivity analysis is provided to identify the links that have the greatest impact on the network performance.

The derivatives of the whole road network’s unblocked reliability with respect to random link’s unblocked reliability are given by the following equation:

\[
S_x = \frac{\partial R}{\partial R_a} \tag{8}
\]

3. ROUTE CHOICE PATTERN

To estimate the unblocked reliability, a route choice pattern which satisfies the UE criterion must be calculated first. The route choice pattern is comprised of routes selected by drivers (road network users) and the route choice proportion on each used route between each OD pair.

3.1 Solving for user equilibrium

User Equilibrium (UE) is the state in which there is no motivation to change the system. UE means that travel times are identical along any used routes connecting a pair of an origin and a destination, and less than or equal to the travel time on all unused routes, if every traveler attempts to choose the path with the shortest travel time. The UE assignment problem is to find the link flows that satisfy the UE criterion when all OD entries have been appropriately assigned. This link flow pattern can be obtained by solving a mathematical program. The flowchart for this calculation is constructed based on the Frank-Wolf algorithm\(^7\) for solving UE as shown in Fig. 1.

3.2 Extended Frank-Wolf algorithm

The traditional Frank-Wolf algorithm gives link flows only. The method of sharing proportion on each route between each OD pair is proposed. At the first iteration, \( k = 1 \), all-or-nothing assignment insures that all OD trip rates are assigned to the network. At the \( k \)th iteration, suppose there is a set of used routes, \( M, m \in M, m = \{1,2,\cdots,w\} \), between a certain OD pair. Implementing step 2 of Fig. 1, the shortest path, \( l \), is found between a certain OD pair. If \( l \) does not belong to the set of \( M \), the loading proportions of each path between the OD pair are expressed as the following equation for the next iteration, \( k = k + 1 \):

\[
\begin{aligned}
   p^{l,k+1} &= \alpha^k, & l &\notin M \\
   p^{m,k+1} &= p^{m,k} (1 - \alpha^k), & m &\in M
\end{aligned} \tag{9}
\]

If \( l \) belongs to \( M \), the loading proportions of the paths between the OD pair are expressed as the following equation for the next iteration, \( k = k + 1 \):

\[
\begin{aligned}
   p^{l,k+1} &= \alpha^k + p^{l,k} (1 - \alpha^k), & l &\in M \\
   p^{m,k+1} &= p^{m,k} (1 - \alpha^k), & m &\notin l \text{ and } m \in M
\end{aligned} \tag{10}
\]
3.3 Carrying out route choice pattern from observed link flows

In the previous section, how to calculate the route choice pattern was analyzed when the OD matrix is known. However, under current conditions the traffic demand can not be obtained directly. Therefore, it is necessary to construct an algorithm to calculate the route choice by the observed link flows that can be surveyed easily by sensors or traffic surveys. The path flow is expressed as a function of the OD flow and loading proportion as:

$$f_{ij}^* = p_{ij}^* q_{ij}$$  \hspace{1cm} (11)

Eq. (11) also shows the function which can be used in projections of traffic demand matrix by link flows. It explains that the sum of the estimated OD flow multiplying the route choice proportion of the OD flow using link $a$ should be consistent with the observed traffic volumes for link $a$. There is multi-path proportion algorithm\(^8\) to estimate the OD matrix based on observed link flows. The loading proportion of each used path is given by the extended Frank-Wolf algorithm in the iteration process of a multi-path proportion algorithm.

4. LOCAL ROAD NETWORK ANALYSIS

The trunk road network of a local area is employed as a case study. Fig. 2 shows the topology of the trunk road network. There are a national road and a parallel motorway in the north-south direction, three national roads and some collector-distributor roads. The nodes from 1st to 11th express traffic zones which are the centroid of studied cities. The total observed traffic flows are considered to occur at the intercity. From the 12th node to the 21st node expresses the interchange of the motorway. The 22nd node is an important intersection between two national roads.

The observed link flows and the parameters of the local area trunk road network are calculated by the traffic census data in 1999\(^9\).

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**Fig. 1** Flowchart of Frank-Wolf algorithm for solving UE

**Fig. 2** Trunk road network of local area
4.1 Unblocked reliability analysis

OD pair’s unblocked reliability is calculated using Eq. (6). The OD pairs’ reliability is shown by the three-dimensional cone chart in Fig. 3, where the horizontal axis (abscissa) is the code of origin traffic zone, the vertical axis (ordinate) is the code of destination traffic zone, and the cone height indicates the reliability level. Based on Eq. (7), the unblocked reliability of the whole road network is calculated and results in 55%. The operation performance of the road network is judged to be in medium congestion. Fig. 4 shows the OD pairs’ reliability when the traffic demand increases 50%. In Fig. 4, the reliability of OD pairs diminishes significantly, the whole network reliability falls to 35%, and the road network is seriously congests. By comparing Figs. 3 and 4, the proposed new method can effectively evaluate the operation performance of the road network at several traffic demand levels.

4.2 Sensitivity calculation

Based on Eq. (8), the sensitivities of links are calculated. The first 10 links with the highest sensitivities are shown in descending order in Table 1. In this table, the link with the greater sensitivity means a more important link. In the present study, the important links have been found, such as the links listed in Table 1. The unblocked reliability of the whole road network will thus be increased greatly if the capacities of the important links are increased.

<table>
<thead>
<tr>
<th>Order</th>
<th>Link</th>
<th>Sensitivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10-7</td>
<td>0.088</td>
</tr>
<tr>
<td>2</td>
<td>8-16</td>
<td>0.081</td>
</tr>
<tr>
<td>3</td>
<td>3-4</td>
<td>0.078</td>
</tr>
<tr>
<td>4</td>
<td>9-20</td>
<td>0.073</td>
</tr>
<tr>
<td>5</td>
<td>20-9</td>
<td>0.050</td>
</tr>
<tr>
<td>6</td>
<td>4-3</td>
<td>0.048</td>
</tr>
<tr>
<td>7</td>
<td>16-8</td>
<td>0.034</td>
</tr>
<tr>
<td>8</td>
<td>10-11</td>
<td>0.031</td>
</tr>
<tr>
<td>9</td>
<td>2-3</td>
<td>0.031</td>
</tr>
<tr>
<td>10</td>
<td>7-22</td>
<td>0.029</td>
</tr>
</tbody>
</table>

5. CONCLUSION

For solving the route choice pattern in the calculation of unblocked reliability, the extended Frank-Wolf algorithm is applied, which result in link flows, selected routes and loaded proportion of each route used. A new methodology is provided to assess the operation performance using unblocked reliability under the User Equilibrium state. As a result, the operation performance of the road network can be evaluated effectively at several levels of traffic demand, and the sensitive links for operation performance can be found in order to direct planning of a road network.

REFERENCES