Interpretation of Cs-corrector System with Twin Hexapoles and Transfer Doublets based on Geometrical Optics Theory

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Abstract: Optical properties of aberration correction systems for electron microscopes, called “Cs-corrector”, were considered from the point of view of the geometrical optics theory. Here, we focused on the Cs-corrector consisted of twin hexapoles and two round lenses. Although the above system has been already theoretically investigated and successfully developed, we have tried to interpret its properties based on the geometrical optics to understand them more intuitively. By using it, electron trajectories could be easily traced and effects of aberrations on the Gaussian image plane were analytically calculated. Analyses of the aberrations up to 5th order provide us that hexapoles and round lenses could eliminate up to 3rd order aberrations in the total optical system including the objective lens, when these 4 elements are properly arranged. The 4th and 5th aberrations were also able to suppress by adding 2 round lenses. These studies were quite significant for designing the aberration correction system actually.

Keywords: Aberration correction, Cs-corrector, Geometrical optics theory, Electron microscope, Hexapole

1. INTRODUCTION

Correction of the spherical aberration is indispensable to achieve sub-angstrom resolution in the electron microscopes. Recently, some types of aberration correction devices consisted of multi-pole lenses (so called “Cs-corrector”) had been successfully developed and have been put to practical use [1-3]. One of the accomplished systems is so called “Rose-type Cs-corrector” consisted of two identical hexapoles (Hex1 and 2) [4-6] connected by two round lenses (transfer lens doublet; TL, each focal length is \( f \)). Most important point is that these four lenses are arranged at particular distances, \( f \), \( 2f \), and \( f \), resulting in the corrector length of \( 4f \), so called “4\( f \)-system”(see Fig.1(a)). Such an arrangement yields no 2nd order aberration and 3rd order aberration of negative spherical aberration coefficient. In the present paper, we report properties of the above system up to 5th order aberrations based on the geometrical optics, according to Crewe’s theory [4].

2. METHOD

Figs.1 show schematic diagram of Cs-corrector optical systems incorporated in the scanning transmission electron microscope (STEM). Here, an electron beam cross-over (a point at the left side of each image), an objective lens (Obj) having just 3rd order spherical aberration and the Gaussian image plane (a line perpendicular to optical axis drawn at the right side of each image) were only considered in the STEM optical system. The elements of the corrector were added into this system, and residual aberrations were investigated at each stage (Figs. 1(a) - (c)). The electron beam trajectories were analytically calculated as follows;

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(1) Electrons go straight ahead in the vacuum space.
(2) Trajectory alterations in the hexapoles are calculated based on the motion equation in the magnetic field. It is approximately limited up to 5th order of “position” and “inclination” of the electron trajectory at the entrance plane of the Hex1.
(3) Hexapole magnetic field along the optical axis is assumed to be constant, and to have rectangular-shape distribution with no leakage outside the hexapoles.
(4) Magnetic fluxes are assumed to be completely contributed to form the hexapole field with no saturation in the cores and no skipping to the neighbor cores.
(5) Round lenses except the objective lens have only focusing effect with no aberrations.

Finally, the position on the Gaussian image plane, where the electron trajectory reached, was determined. Difference between it and the point across the optical axis corresponds to the aberrations.

3. RESULTS AND DISCUSSIONS

The hexapole magnetic field on the plane perpendicular to the optical axis has trigonal symmetry as show in Fig. 2. It is therefore easily understood that the electron trajectories contain trigonally-symmetric component, which corresponds to the 2nd order term. The 3rd order rotationally-symmetric component to cancel out the spherical aberration of the objective lens is also produced simultaneously, but the 2nd order one is here dominant. To delete the 2nd order component, another hexapole identically excited is required. When these two hexapoles are connected by incorporating round lenses between them, which act to image the central plane of the Hex1 into that of the Hex2 as one-to-one transferring, the 2nd order one produced by the Hex1 is transferred into the Hex2 with turn by 180 degrees, resulting in counteraction of the 2nd order trigonally-symmetric component. As mentioned above, two hexapoles and two round lenses must be set with a distance of $f$, $2f$, and $f$ so that they achieve one-to-one transferring. In the case of Fig. 1(a), the position of the electron trajectory on the image plane $(x_0, y_0)$ was mathematically derived. The following equation (1) show only $x_i$ because $y_i$ is easily derived by modifying this equation.

$$
\begin{aligned}
x_i &= \left[ -k E L M \left( \frac{2}{7} + \frac{4}{7} y + \frac{4}{3} y^2 + \frac{2}{11} y^3 + \frac{4}{23} y^4 + \frac{32}{35} y^5 - \frac{1}{5} y^6 \right) + \frac{Cs (1 + 3y^2)}{M^2} \right] \times r_i' \cos \theta \\
&+ \left[ k E L M \left( \frac{1}{38} + \frac{9}{112} y + \frac{25}{1080} y^2 + \frac{5}{2016} y^3 \right) \right] \times r_i' \cos \theta \left( \cos 4\theta + 4 \cos 2\theta \right) \\
&+ \left[ k E L M \left( \frac{3}{2} y^2 + \frac{22}{23} y^3 + \frac{2}{7} y^4 + \frac{2}{11} y^5 \right) \right] \times r_i' \left( \cos 4\theta + \cos 5\theta \right) \\
&+ \left[ \frac{Ck E L M (1 + 3y^2)}{140} \right] \left[ \left( \frac{1}{3} + \frac{1}{2} y + \frac{1}{14} y^2 \right) + \frac{32}{35} \left( \frac{2}{3} y + \frac{2}{5} y^2 + \frac{2}{11} y^3 + \frac{2}{18} y^4 \right) \right] \times r_i' \cos \theta \\
&+ \left[ k E L M \left( \frac{2}{42} + \frac{2}{14} \right) \right] r_i' \left( 4 \cos \theta - 3 \cos 6\theta \right) \\
&= 0
\end{aligned}
$$

Here, the position and the inclination of the electron trajectory on the entrance plane of the Hex1 are $(x_0, y_0)$, $(x_0', y_0')$, respectively. The sum of their multiplier factor indicates the “order” of the aberration, as mentioned above.

$k$ corresponds to excitation intensity of the hexapole. $L$ is length of the hexapole along the optical axis. $M$ and $Cs$ are magnification and spherical aberration coefficient of the objective lens, respectively. In addition, some transformations of variables were done as:

$$
(x_0, y_0) = (r_0 \cos \theta, r_0 \sin \theta), S = x_0' / \gamma_0', y_0' = y_0 / \gamma_0'.
$$

The position $(x_0, y_0)$ is rewritten by the polar coordinate system. Since the variable $S$ and $\gamma$ have no influence on the order of the aberration, it is determined by only multiplier factor of $r_0$. In the equation (1), it can be confirmed that the 2nd order term is successfully canceled out. The 1st term corresponds to 3rd order aberration with rotationally symmetry because of $r_0^3$ and $\cos \theta$. Note that this term can be zero if the hexapole excitation intensity $k$ is selected as the appropriate value, meaning that the spherical aberration of the objective lens can be corrected by using the $4f$-system $Cs$-corrector. However, 4th and 5th order aberrations (the other terms in equation (1)) are remained.

The 4th order one vanishes when the electron trajectories into the Hex1 are parallel to the optical axis. In this case, $M$ and $\gamma$ become zero. Therefore, the 2nd and 3rd terms in equation (1) also become zero, resulting in the correction of the 4th order aberration. To achieve it, a round lens in front of the Hex1 (AL1; Alignment Lens 1) should be arranged (see Fig.1(b)). At this stage, the residual aberrations are then 5th order, which are rotationally symmetric and hexagonally-symmetric components (4th and 5th terms in equation (1), respectively). The former is almost proportional to the distance between the Hex2 and an objective lens, $T$. It cannot be therefore ignored in the ordinary STEM system because of large $T$ by existing the scanning coils between them.

The 5th order aberrations can be minimized by incorporation of an additional round lens behind the Hex2 (AL2; Alignment Lens 2, focal length is $f_{AL2}$) and making a crossover between Hex2 and the objective lens at a particular position, as shown in Fig.1(c). The equation corresponding to (1) at this stage is derived as;

$$
\begin{aligned}
x_i &= \frac{-2k E L M f_{AL2}}{3} \left( -\frac{Cs (1 + 3y^2)}{M^2} \right) \times r_i' \cos \theta \\
&+ \left[ \frac{3k E L M}{f_{AL2}} \left( \frac{1 + 3y^2}{M^2} \right) \left( \frac{2}{3} a + \frac{2}{18} \right) \right] r_i' \cos \theta \\
&+ \left[ \frac{1}{42} k E L M \right] r_i' \left( 4 \cos \theta - 3 \cos 6\theta \right).
\end{aligned}
$$

This indicates that the 3rd order aberration can be corrected as well as the above case, however, the required
value of $k$ is different. The 5th order rotationally symmetric aberration (2nd term in equation (2)) can be eliminated by setting a proper optical condition, that is;

$$f_{at} + a + \frac{aT}{f_{at}} - \frac{aL}{2f_{at}} = 0$$

Here, $a$ is the distance between the objective lens and the cross-over position in front of it. This actually corresponds to a condition that the AL2 images the central plane of the Hex2 into the objective lens, meaning that the distance $T$ between them become zero equivalently.

Then, the final term in the equation (2) is still remained. At the present, we have not established a method to compensate it completely. However, a method to minimize its effect is found. When the AL2 condition is slightly deviated from that for satisfying the equation (3), arising rotationally-symmetric component and the residual hexagonally-symmetric one can be counteracted with each other to a certain extent, because of their opposite sign.

4. CONCLUSION

The aberrations on the Gaussian image plane was analytically calculated by using the geometrical optics theory in the case of the STEM equipped with the 4f-system Cs-corrector. Even by incorporating only the Cs-corrector, the aberrations up to 3rd order could be corrected. The compensation of the 4th order aberration required an additional round lens in front of the Hex1 to make the incident electron trajectories parallel to the optical axis. The 5th order rotationally-symmetric aberration could be eliminated by adding a round lens between the Hex2 and the objective lens, however, 5th order hexagonally-symmetric aberration was still remained. To minimize it, a proper degree of rotationally-symmetric one was left on purpose and counteracted the residual aberration. Based on the above results, we can specifically design the Cs-corrector system by substituting actual parameters of the STEM apparatus.

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